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Our search has been less for phenomena to "explain" than for non-trivial magnetohydrodynamic processes in the fully turbulent domain that can be understood. These processes are certain to be present in a great many situations, but seldom in isolation from all other processes which could mask

or distort them. Two immediate examples will illustrate: (1) development of anisotropy in the presence of a strong dc magnetic field; and (2) the decay of an MHD turbulent field to a dynamically aligned state with velocity field and magnetic fields parallel or antiparallel ("Alfvénic" fluctuations). Both effects are known to occur in real plasmas under a variety of circumstances, and we have been at work on both. The first we claim to have explained satisfactorily within the last year, the second we have not.

(1) Development of Anisotropy [15]*

It has been known for some time for fluctuating magnetic fields in turbulent situations involving a mean dc magnetic field, the fluctuations are anisotropic, with length scales perpendicular to the mean $\langle B \rangle$ considerably shorter than those parallel to it. (This anisotropy in a laboratory context may also be used to justify a derivation of the "Strauss equations" of strongly anisotropic MHD.) The origin of the anisotropy had been mysterious until extensive numerical computations by Shebalin, Matthaeus, and Montgomery [15]* suggested a surprisingly simple analytical mechanism: an inhibition of spectral transfer in the parallel wavenumber direction relative to that in the perpendicular direction, due to the kinematic constraint that in a resonant triad of interacting incompressible Alfvén waves, at least one of the three has to have zero parallel wavenumber. Transfer at this order is not possible in the parallel direction but is possible in the perpendicular direction, and proceeds to broaden the k_{\perp} spectrum until it encounters dissipation scales. Thus the degree of anisotropy increases as the Reynolds numbers increase, but

* Reference numbers here and elsewhere in this section refer to the collection of abstracts at the end of this Section.

exhibits only a weak dependence on the external magnetic field strength after the dc field passes two or three times the rms fluctuating field strength. The mechanism is so simple that one wonders why it had to be suggested by numerical evidence, but the example stands as an impressive illustration of how in nonlinear mechanics, thought often flows from action, in the form of numerical solution of intractable nonlinear equations.

(2) Alignment of \mathbf{v} and \mathbf{B} Fields [3]

The high degree of alignment or anti-alignment of \mathbf{v} and \mathbf{B} (velocity and magnetic) fields often observed in the turbulent solar wind has been seen both in a statistical closure calculation (Grappin et al, Astronomy and Astrophysics 105, 6 (1982)) and in numerical solution of the MHD equations [3]. The basic physical reason for the alignment is still unknown, however: whether the alignment is a consequence of the way the turbulence is produced close to the sun, or whether it is a result of a transfer process, is still debatable. We believe it to be the latter, but extensive computations have not settled the matter, and the topic is proposed for further research in Section III of this proposal. The complication is that the "selective decay," which we postulate as the origin of the alignment, competes with another selective decay process which has a different final state as its direction of evolution [1,13], and which has also been documented under some circumstances. The phase space boundary between the regimes of the two processes, if there is one, has been difficult to identify. The problem stands, at this moment, as an unresolved theoretical challenge.

Other areas of research activity during the past year include the following.

(3) Asymptotic Long-Time State of an Inverse Cascade [19]

The inverse cascade is a by-now familiar phenomenon in which a global invariant quantity supplied at short wavelengths to a turbulent field can, through inverse spectral transfer, cascade to long wavelengths and generate coherent structures of dimensions much larger than the injection scale. Theoretical treatments have conventionally assumed an unbounded system with arbitrarily many additional octaves at the lower end of wavenumber space to expand into. Real plasmas and computer simulations always have a maximum wavelength (minimum wave number), however, because they are of a finite size. It is unclear as to what should occur when the longest wavelengths in the system "fill up" to a level of excitation compatible with the predicted power law behavior. Does the inverse cascade cease? Does the entire spectrum between the injection scale and the maximum length scale rise together? Or does the fundamental (longest wavelength) run off and leave the rest of the spectrum, continuing to absorb the inversely cascaded quantity? Still other scenarios might be imaginable, but it seems to have been definitively established, by a series of extensive, forced-turbulent computations, that it is the third one which occurs, both for Navier-Stokes flow and for magnetohydrodynamics (Hossain, Matthaeus, and Montgomery [19]). The fundamental at very long times comes ultimately to dominate the spectrum and is limited by its own internal dissipation. A semi-quantitative algebraic model of the final saturated state has had some limited success in predicting the computed results.

(4) Thresholds for the Onset of Tearing-Mode Turbulence in the Presence of Finite Viscosity and Resistivity [17]

The most ubiquitous source of MHD turbulence, in space and in fusion devices, seems to be the presence of non-uniform electric currents. Most of

our work over the last several years has involved computational investigations into fully developed turbulence at the highest (affordable) Reynolds numbers, but within the last year we have also developed an interest in the transition regime. The transition boundary in a quiescent, current-carrying magnetofluid is governed by the Lundquist number S and its viscous analogue, M , in an inseparable combination \sqrt{MS} . Since most investigations of incompressible tearing modes have heretofore involved finite S but infinite M ("resistive" but "non-viscous" calculations), we have seen it as imperative to carry out investigations for both M and S finite. This increases the order of the differential equations in the relevant eigenvalue problem and defines a formidable time-independent two-point boundary-value problem [17]. We have chosen to solve it by adapting the spectral-method expansion in Chebyshev polynomials used by Orszag [J. Fluid Mech. 50, 689 (1971)] to solve the Orr-Sommerfeld equation to six-figure accuracy. This has proved exceedingly useful, and it is our intent to continue these transition calculations into the nonlinear regime, as described further in Section III.

(5) Other Activities over the Past Year

Three-dimensional spectral-method codes for solving the Strauss equations [13] of dissipative 3D MHD have been written by W. H. Matthaeus, and are currently being run largely as a result of a very generous allotment of CRAY-1 time from the Department of Energy's MFE program. We have yet to generate our first manuscript on this subject, but it is currently our most active area of activity (see Section III).

Numerous invited talks have been given by the Principal Investigator during the last year: (1) two talks in Sweden, one at the International

Conference on Plasma Physics [13] and one at the Chalmers Symposium on Plasma Theory [14]; (2) a talk at the ICASE Symposium on Spectral Method Solution of Partial Differential Equations, NASA Langley Research Center [16]; (3) a talk at "Solar Wind Five" [18], reviewing the present state of MHD turbulence Physics; (4) invited seminars and/or colloquia at Lehigh, Columbia, Los Alamos, Arizona, and the University of Texas, Austin. As indicated in the list of abstracts, several of these will lead to publications and are already in press.

The writer has also arranged a symposium of four invited talks on "MHD Turbulence in Space" [Parker, Goldstein, Matthaeus, Gekelmann] for the spring AGU meeting in Baltimore.

During the last year, the writer has been elected to the Executive Committee of the Division of Plasma Physics of the American Physical Society.

Currently the writer is supervising four Ph.D. candidates' research in plasma theory at William and Mary, and is getting ready to begin a four-month stint as Visiting Professor at the Institute of Plasma Physics in Nagoya, Japan.

W. H. Matthaeus [20] also gave a paper at "Solar Wind Five" which will be published.

PAPER PUBLISHED AND/OR SUBMITTED SINCE JANUARY, 1981 (ABSTRACTS)

- (1) W. H. Matthaeus and D. Montgomery, "Why Should Energy Decay While Magnetic Helicity is Conserved?: The Essentials of Turbulent Spectral Transfer". Proc. Reversed-Field Pinch Theory Workshop, H. R. Lewis, Editor. Los Alamos Report LA-8944-C Conference, UC-20g (1982).

[No Abstract]

- (2) S. Riyopoulos, A. Bondeson, and D. Montgomery, "Relaxation Toward States of Minimum Energy in a Compact Torus", Phys. Fluids 25, 107 (1982).

A finite-difference, resistive, magnetohydrodynamic code is used to follow the long-time evolution of decaying nonequilibrium states inside a rigid, perfectly conducting cylindrical boundary. The energy-to-magnetic helicity ratio decays toward a minimum, in accord with a conjecture of Taylor. The magnetic Reynolds number is considerably higher than the mechanical Reynolds number for the regime considered. The energy, which is mostly magnetic, tends to decay in bursts associated with current filamentation and magnetic reconnection.

- (3) W. H. Matthaeus, M. L. Goldstein, and D. Montgomery, "Dynamic Alignment of Velocity and Magnetic Fields in Magneto Hydrodynamic Turbulence", EOS (Transactions of the AGU) 63, SS32-10 (1982).

The idea that hydromagnetic turbulence preferentially leads to states of aligned velocity and magnetic fields dates to the work of Chandrasekhar¹. More recently, computer simulations² and closure calculations³ have strengthened this conjecture. Here we report simulation results, in a 2-dimensional geometry in which the mean magnetic field plays no role, which indicate that: 1) MHD turbulence usually causes the absolute value of H_c/ϵ to increase in time, where $H_c = \langle \underline{V} \cdot \underline{B} \rangle$ and the energy $\epsilon = \langle \underline{V}^2 + \underline{B}^2 \rangle$. \underline{V} is the fluid velocity and \underline{B} the magnetic field in Alfvén speed units. 2) The system evolves to a turbulent state even when initial conditions are very "near" the stationary state for which $\underline{V} = \underline{B}$ at all points. It is suggested that nearly "Alfvénic" states in the solar wind plasma may not prohibit active turbulent processes as much as has been previously thought.

¹ S. Chandrasekhar, Proc. Nat. Acad. Sci. 42, 273 (1956).

² J. Leorat, R. Grappin, A. Pouquet and U. Frisch, Observatoire de Nice preprint (1980).

³ M. Dobrowolny, A. Mangeney and P. Veltri, Phys. Rev. Lett., 45, 144 (1980).

- (4) W. H. Matthaeus and D. Montgomery, "Nonlinear Evolution of the Sheet Pinch", J. Plasma Phys. 25, 11 (1981).

An incompressible, dissipative numerical code of the spectral type is used to follow the nonlinear evolution of a magnetohydrodynamic sheet pinch in two spatial dimensions. The evolution involves considerable turbulent activity in the electric current field, with the excited spatial scales ranging from the size of the containing volume down to the dissipation lengths of the

magnetic and velocity fields. Strong current filamentation near magnetic X-points is observed, as is 'jetting', or expulsion of magnetofluid from the vicinity of the X-point parallel to the current sheet.

- (5) J. Ambrosiano and G. Vahala, "Most Probable Magnetohydrodynamic and Reversed Field Pinch Equilibria", *Phys. Fluids* 24, 2253 (1981).

The statistical theory of Montgomery, Turner, and Vahala, which determines the most probable ideal magnetohydrodynamic equilibrium compatible with given information on only a few global parameters (e.g., energy E , magnetic Helicity H , flux Φ , current I , ...) is extended and investigated for both the tokamak regime (in which experimentally $\Phi \gg \mu_0 a I$, with a being the plasma radius) and the reversed field pinch regime ($\Phi \ll \mu_0 a I$). One obtains typical experimentally relevant profiles in the appropriate regimes. Most probable equilibria sequences are investigated as the energy/magnetic helicity ratio is decreased at fixed flux and current: In the tokamak regime (flux \gg current) the diamagnetic toroidal field B_z becomes less diamagnetic and tends to a uniform field, while in the reversed field pinch regime (flux \ll current), field reversal sets in B_z with the radial reversal position moving farther into the plasma and the eventual appearance of hollow pressure profiles. It appears that, in both regimes, the most probable equilibria are becoming more stable as $\mu_0 a E/H$ decreases. Linearized analytic force-free states can also be constructed for certain regimes of the global parameters together with their nonlinear quasi-force-free numerical counterparts.

- (6) D. Montgomery, "Maximal Entropy in Fluid and Plasma Turbulence: A Review". [To appear in Proc. Workshop on Maximum Entropy Estimation and Data Analysis, Univ. of Wyoming, June 8-10, 1981.]

Three recent applications of maximal entropy procedures to models of turbulence in plasmas are described. They are: (1) the calculation of "most probable" states in guiding-center plasmas and vortex fluids; (2) the calculation of "most probable" magnetohydrodynamic equilibrium profiles; and (3) the prediction of absolute equilibrium Gibbs ensemble spectra for Navier-Stokes fluids. Further conjectures on the role of entropies in turbulence theories are offered.

- (7) D. W. Swift, "Numerical Simulation of the Generation of Electrostatic Turbulence in the Magnetotail", *J. Geophys. Res.* 86, 2773 (1981).

A two-dimensional plasma model is used to investigate the development of electrostatic turbulence in a magnetized plasma from plasma instabilities. The simulation consists of following the motion of 10^5 ions in their self-consistent electrostatic field. The electrons are treated as a constant neutralizing background. The instabilities modeled are driven by a ring-type velocity distribution and by interpenetrating ion beams in a time variable magnetic field. Instability growth times are of the order of an ion gyroperiod in the case of the ring distribution and of the order of an ion plasma period in the case of the beam simulation. Maximum potential differences generated are of the order of the ion kinetic energies. These simulations demonstrate the cascade of wave energy to long wavelengths, thus showing that $E \times B$ turbulence can be generated from plasma microinstabilities. After the free energy

feeding, the instabilities are exhausted, and wave energy at wavelengths less than an ion gyrodiameter decays quickly to equilibrium levels, while longer wavelength modes persist for much longer times. In one model with a time dependent, but spatially uniform, magnetic field the electric field energy at long wavelengths appeared to increase as a result of the increase of the magnetic field. [This is work begun at William and Mary, and completed at the University of Alaska, and was supported in part by NSG-7416.]

- (8) W. H. Matthaeus, M. L. Goldstein, and C. Smith, "Evaluation of Magnetic Helicity in Homogeneous Turbulence", Phys. Rev. Lett. 48, 1256 (1982).

A technique is presented for the measurement of magnetic helicity from values of the two-point magnetic-field correlation matrix under the assumption of spatial homogeneity. Knowledge of a single scalar function of space, derivable from the correlation matrix, suffices to determine the magnetic helicity. The technique is illustrated by a report of the first measurement of the magnetic helicity of the solar wind.

- (9) D. Montgomery and L. Turner, "Anisotropic Magnetohydrodynamic Turbulence in a Strong External Magnetic Field", Phys. Fluids 24, 825 (1981).

A strong external dc magnetic field introduces a basic anisotropy into incompressible magnetohydrodynamic turbulence. The modifications that this is likely to produce in the properties of the turbulence are explored for the high Reynolds number case. The conclusion is reached that the turbulent spectrum splits into two parts: an essentially two-dimensional spectrum with both the velocity field and magnetic fluctuations perpendicular to the dc magnetic field, and a generally weaker and more nearly isotropic spectrum of Alfvén waves. A minimal characterization of the spectral density tensors is given. Similarities to measurements from the Culham-Harwell Zeta pinck device and the University of California, Los Angeles Macrotron toakmak are remarked upon, as are certain implications for the Belcher and Davis measurements of magnetohydrodynamic turbulence in the solar wind.

- (10) D. Montgomery and L. Turner, "Two-and-a-half-dimensional Magnetohydrodynamic Turbulence", Phys. Fluids 25, 345 (1982).

Incompressible magnetohydrodynamic turbulence is considered in a geometry in which the fields are all independent of the z coordinate, e.g., but have all three components. This is a limit frequently called "two-and-a-half-dimensional." It is conjectured that simultaneous inverse cascades of magnetic helicity and mean square z -component of vector potential may exist. At high Reynolds numbers, selective decays to states of finite pressure gradients appear to be possible. Anti-dynamo theorems are difficult to prove for this geometry.

- (11) W. H. Matthaeus and C. Smith, "Structure of Correlation Tensors in Homogeneous Anisotropic Turbulence", Phys. Rev. A 24, 2135 (1981).

We extend the theory of isotropic tensors, developed by Robertson, Batchelor, and Chandrasekhar, to cover the general case of turbulence with a

pseudovector-preferred direction, without assuming mirror-reflection invariance. Attention is focused on two-point-correlation functions, and it is shown that the form of the decomposition into proper and pseudotensor contributions is restricted by the homogeneity requirement. We present the somewhat unexpected result that the vector- and pseudovector-preferred-direction cases yield different results: A pseudovector-preferred direction allows the correlation matrix one more functional degree of freedom than does the "proper" vector case. We present an explicit form of the two-point-correlation tensor in the presence of a uniform mean magnetic field which may be appropriate for use in analysis of amgnetic fluctuations in plasma containment devices or the interplanetary medium. A procedure for determining the magnetic helicity from experimental data is presented.

(12) D. Montgomery, "The Computation of Inverse Magnetic Cascades", Proc. U.S.-Japan Workshop on 3D MHD Studies, B. A. Carreras, Editor, Oak Ridge, CONF-811 0101 (1982).

Inverse cascades of magnetic quantities for turbulent incompressible magnetohydrodynamics are reviewed, for two or three dimensions. The theory is extended to the Strauss equations, a description intermediate between two and three dimensions appropriate to tokamak magnetofluids. Consideration of the absolute equilibrium Gibbs ensemble for the system leads to a prediction of an inverse cascade of magnetic helicity, which may manifest itself as a major disruption. An agenda for computational investigation of this conjecture is proposed.

(13) D. Montgomery, "Major Disruptions, Inverse Cascades, and the Strauss Equations", [Invited paper presented at the International Conference on Plasma Physics, Göteborg, Sweden, June 9-15, 1982. Physica Scripta T2 1982, pp. 83-88.

Current-carrying plasmas in a strong dc magnetic field are subject to violent disruptions above certain thresholds. At present difficult to verify, explanations are typically sought in terms of "tearing modes". An alternative explanation is in terms of inverse magnetic helicity cascades, generated from a variety of possible sources of small-scale MHD turbulence. Strongly anisotropic MHD plasmas may be described by the Strauss equations. Indications of turbulent inverse cascade behavior for the Strauss equations are sought, in parallel with earlier examples from MHD and fluid mechanics.

(14) D. Montgomery, "Thresholds for the Onset of Fluid and Magnetofluid Turbulence", [Invited paper presented at the Chalmers Symposium on Plasma Theory and Experiment, Aspenåsgården, Sweden, June 16-18, 1982. Physica Scripta T2 1982, pp. 506-510.

Hydrodynamic stability theory has focussed on a few simple test cases to obtain the sharpest possible confrontations between theory and experiment. Six of these are briefly reviewed: plane Poiseuille and Couette flow, pipe flow, rotating Couette flow, thermally-driven Bénard convection, and the Blasius laminar boundary layer. Linear perturbation theory seems inadequate in the first three cases, and satisfactory in the last three. Insufficient information, experimental or theoretical, exists in magnetohydrodynamics to make any comparably decisive tests.

- (15) J. V. Shebalin, W. H. Matthaeus, and D. Montgomery, "Anisotropy in MHD Turbulence Due to a Mean Magnetic Field", (J. Plasma Phys., in press, to appear in 1983).

The development of anisotropy in an initially isotropic spectrum is studied numerically for two-dimensional magnetohydrodynamic turbulence. The anisotropy develops due to the combined effects of an externally imposed dc magnetic field and viscous and resistive dissipation at high wave numbers. The effect is most pronounced at high mechanical and magnetic Reynolds numbers.

- (16) D. Montgomery, "Applications of Spectral Methods to Turbulent Magnetofluids in Space and Fusion Research," invited paper given at ICASE Symposium on Spectral-Method Solution of Partial Differential Equations, NASA Langley Research Center, Hampton, Virginia, Aug. 16-18 (1982); to be published in 1983 in Proceedings, by Society for Industrial and Applied Mathematics, Philadelphia, PA.

Recent and potential applications of spectral method computation to incompressible, dissipative magnetohydrodynamics are surveyed. Linear stability problems for one-dimensional, quasi-equilibria are approachable through a close analogue of the Orr-Sommerfeld equation. It is likely that for Reynolds-like numbers above certain as-yet-undetermined thresholds, all magnetofluids are turbulent. Four recent effects in MHD turbulence are remarked upon, as they have displayed themselves in spectral method computations: (1) inverse cascades; (2) small-scale intermittent dissipative structures; (3) selective decays of ideal global invariants relative to each other; and (4) anisotropy induced by a mean dc magnetic field. Two more conjectured applications are suggested. All the turbulent processes discussed are sometimes involved in current-carrying confined fusion magnetoplasmas and in space plasmas.

- (17) R. B. Dahlburg, T. A. Zang, D. Montgomery and M. Y. Hussaini, "Viscous, Resistive, MHD Stability by Spectral Techniques." Submitted to Proc. Nat. Acad. Sci. (USA), 1983.

Expansions in Chebyshev polynomials are used to study the linear stability of one-dimensional magnetohydrodynamic (MHD) quasi-equilibria, in the presence of finite resistivity and viscosity. The method is modeled on the one used by Orszag in accurate computation of solutions of the Orr-Sommerfeld equation. Two Reynolds-like numbers involving Alfvén speeds, length scales, kinematic viscosity, and magnetic diffusivity govern the stability boundaries, which are determined by the geometric mean of the two Reynolds-like numbers. Marginal stability curves, growth rates versus Reynolds-like numbers, and growth rates versus parallel wave numbers are exhibited. A numerical result which appears general is that instability has been found to be associated with inflection points in the current profile, though no general analytical proof has emerged. It is possible that nonlinear subcritical three-dimensional instabilities may exist, similar to those in Poiseuille and Couette flow.

- (18) D. Montgomery, "Theory of Hydromagnetic Turbulence," invited paper given at "Solar Wind Five," Woodstock, Vermont, Nov. 1, 1982; to be published in Proceedings.

The present state of MHD turbulence theory as a possible solar wind research tool is surveyed. The theory is statistical, and does not make statements about individual events. It is unreasonable to expect ever to be able to "explain" individual events with turbulence theory. The ensembles considered typically have individual realizations which differ qualitatively, unlike equilibrium statistical mechanics. Most of the theory deals with highly symmetric situations; most of these symmetries have yet to be tested in the solar wind. The applicability of MHD itself to solar wind parameters is highly questionable; yet it has no competitors, as a potentially comprehensive dynamical description. The purposes of solar wind research require sharper articulation. If they are to understand radial turbulent plasma flows from spheres, laboratory experiments and numerical solution of equations of motion may be a cheap alternative to spacecraft. If "real life" information is demanded, multiple spacecraft with variable separation may be necessary to go further. The principal emphasis in the theory so far has been on spectral behavior for spatial covariances in wave number space. There is no respectable theory of these for highly anisotropic situations. A rather slow development of theory acts as a brake on justifiable measurement, at this point.

- (19) M. Hossain, W. H. Matthaeus, and D. Montgomery, "Long-time States for Inverse Cascades in the Presence of a Maximum Length Scale," submitted to J. Plasma Phys., 1983.

It is shown numerically, both for the two-dimensional Navier-Stokes (guiding-center plasma) equations and for two-dimensional magnetohydrodynamics, that the long-time asymptotic state in a forced inverse-cascade situation is one in which the spectrum is completely dominated by its own fundamental. The growth continues until the fundamental is dissipatively limited by its own dissipation rate.

- (20) W. H. Matthaeus and M. L. Goldstein, "Magnetohydrodynamic Turbulence in the Solar Wind," (to appear in Proceedings of "Solar Wind Five," Woodstock, Vermont, Nov. 1-5, 1982).

Recent work in describing the solar wind as an MHD turbulent fluid has shown that the magnetic fluctuations are adequately described as time stationary and to some extent as spatially homogeneous. Spectra of the three rugged invariants of incompressible MHD are the principal quantities used to characterize the velocity and magnetic field fluctuations. Unresolved issues concerning the existence of actively developing turbulence are discussed.